

Macroeconomics and Financial Markets

Asset Pricing

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Outline

- 1 Log-linearization and the Sharpe ratio
- 2 Habit formation
- 3 Epstein-Zin
- 4 Disasters

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Asset Pricing

- Asset pricing equation

$$1 = E_t[M_{t+1}R_{t+1}]$$

$$\text{(often:)} \quad M_{t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t}$$

where λ_t = marg. util. of cons. at t = Lagrange multiplier.

- Nonstochastic steady state:

$$1 = \beta \bar{R}$$

- How about taking into account the steady-state risk premium?

Asset Pricing: joint log-normality



$$1 = E_t[M_{t+1}R_{t+1}] \quad (1)$$

- Notation. $m_{t+1} = \log M_{t+1}$, etc. \hat{M}_{t+1} : log-deviation. Assume (approximate with) joint log-normality.

$$\begin{aligned} 0 &= \log(E_t[\exp(m_{t+1} + r_{t+1})]) \\ &= E_t[m_{t+1} + r_{t+1}] + \frac{1}{2} \left(\sigma_{m;t}^2 + 2\text{cov}_t(m_{t+1}, r_{t+1}) + \sigma_{r;t}^2 \right) \end{aligned} \quad (2)$$

where $\sigma_{m;t}^2 = E_t[(m_{t+1} - E_t[m_{t+1}])^2]$, $\sigma_{r;t}^2 = \dots$ and using

$$X \sim \mathcal{N}(\mu, \sigma^2) \rightarrow \ln(E[e^X]) = \mu + \frac{\sigma^2}{2}$$

Asset Pricing: theory 2

$$0 = E_t[m_{t+1} + r_{t+1}] + \frac{1}{2} \left(\sigma_{m;t}^2 + 2\text{cov}_t(m_{t+1}, r_{t+1}) + \sigma_{r;t}^2 \right)$$

- Risk-free rate. Assume homoskedasticity. Notation: $\sigma_m^2, \sigma_r^2, \rho_{m,r}$ are conditional variances and correlations.

$$r_t^f = E_t[-m_{t+1}] - \frac{1}{2}\sigma_m^2 \quad (3)$$

- Define the **Sharpe Ratio**

$$\mathcal{SR}_t = \frac{\log E_t[R_{t+1}] - r_t^f}{\sigma_r}$$

- Result:

$$\mathcal{SR} = -\rho_{m,r}\sigma_m \quad (4)$$

$$\mathcal{SR}^{\max} = \sigma_m \quad (5)$$

- Data: $\mathcal{SR} = 0.3$. Perhaps, $\rho_{m,r} = 0.2$. Then $\mathcal{SR}^{\max} = 1.5$.

Asset pricing: quantities, Mehra-Prescott

- 1 Recall: $\mathcal{SR}^{\max} = \sigma_m$
- 2 If $u = (c_t^{1-\eta} - 1) / (1 - \eta)$, then, $m_{t+1} = \log \beta - \eta \Delta \log(c_{t+1})$ and $\mathcal{SR}^{\max} = \sigma_m = \eta \sigma_c = 0.01 \eta$
- 3 Need $\eta = 30$ for $\mathcal{SR}^{\max} = 0.3$. Need $\eta = 150$ for $\mathcal{SR}^{\max} = 1.5$.
- 4 Suppose: $u = ((c_t \Phi(n_t))^{1-\eta} - 1) / (1 - \eta)$. Then

$$\hat{m}_{t+1} = -\eta \Delta \hat{c}_{t+1} - (1 - \eta) \kappa \Delta \hat{n}_{t+1}$$

- 5 Leisure goes the wrong way, if $\eta > 1$.
- 6 For $\eta > 1$,

$$\begin{aligned} (\mathcal{SR}^{\max})^2 &= \sigma_m^2 = \eta^2 \sigma_c^2 + (1 - \eta)^2 \kappa^2 \sigma_n^2 + 2\eta(1 - \eta) \kappa \text{cov}(\hat{c}_t, \hat{n}_t) \\ &\leq | \eta \sigma_c^2 - (1 - \eta) \kappa \sigma_n |^2 \end{aligned}$$

- 7 Can also write as

$$\left(\frac{\mathcal{SR}^{\max}}{\eta \sigma_c} \right)^2 = 1 - 2(\eta - 1) \kappa \text{corr}(\hat{c}_t, \hat{n}_t) \frac{\sigma_n}{\sigma_c} + \left((1 - \eta) \kappa \frac{\sigma_n}{\sigma_c} \right)^2$$

Calculating risk premia from the RLOM

- RLOM:

$$x_t = Px_{t-1} + Q\epsilon_t,$$

$$0 = E_t[\epsilon_{t+1}] = 0$$

$$\Sigma = E_t[\epsilon_t \epsilon_t']$$

- Find the two entries in x_t for

$$\hat{m}_t = x_{m,t} = P_m x_{t-1} + Q_m \epsilon_t$$

$$\hat{r}_t = x_{r,t} = P_r x_{t-1} + Q_r \epsilon_t$$

- Calculate conditional variances and covariances:

$$\sigma_m^2 = Q_m \Sigma Q_m'$$

$$\text{cov}_t(m_{t+1}, r_{t+1}) = Q_m \Sigma Q_r'$$

$$\sigma_r^2 = Q_r \Sigma Q_r'$$

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1 Log-linearization and the Sharpe ratio

2 **Habit formation**

3 Epstein-Zin

4 Disasters

Non-separable utility functions

- 1 **Habit formation and “catching-up-with-the-Joneses”.**
- 2 Epstein Zin.

Habit formation preferences



$$U = E \left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right] \quad (6)$$

where

$$u(c_t, l_t) = \frac{((c_t - H_t)(A + (l_t - F_t)^\nu))^{1-\eta} - 1}{1 - \eta} \quad (7)$$

c_t : consumption, l_t : leisure. Parameters β, η, ν, A . Need $\nu > 0$ and $\eta > \nu/(\nu + 1)$ to assure $u(\cdot, \cdot)$ to be increasing and concave.

- “Exogenous” habits / catching up with the Joneses:

$$H_t = e^\gamma ((1 - \zeta)\chi C_{t-1} + \zeta H_{t-1}) \quad (8)$$

$$F_t = (1 - \phi)\psi L_{t-1} + \phi F_{t-1} \quad (9)$$

C_t and L_t : aggregate levels of consumption and leisure.

Parameter Overview

- Preferences: $(\beta, \eta, \nu, A, \chi, \psi, \zeta, \phi)$, where

- ▶ β : discount factor

| | cons. | leisure |
|-------------------|------------|-----------------|
| curvature | $1 - \eta$ | $\nu(1 - \eta)$ |
| ▶ intercept | - | A |
| share of habit | χ | ψ |
| dynamics of habit | ζ | ϕ |

- Other parameters:

- ▶ $1/\xi$: adjustment cost for capital
- ▶ μ : wage rigidity
- ▶ σ_ϵ : TFP volatility.

Constraining Preferences

- - 1 balanced growth condition
 - 2 leisure share: $\bar{L} = 2/3$.
 - 3 Frisch elasticity of labor supply = 3.
 - 4 Steady-state return: $\bar{R} = 1.01$.
 - 5 Sharpe ratio on asset markets = 0.27.
- Strategy:
 - ▶ Find algebraic constraints on the preference parameters...
 - ▶ ... and solve them.

Sharpe ratio

- Assume joint log-normality. Then,

$$\mathcal{SR} = \sigma_\lambda \quad (10)$$

- No habit formation:

$$\left(\frac{\mathcal{SR}^{\max}}{\eta \sigma_c} \right)^2 = 1 - 2(\eta - 1)\kappa \rho_{c,n} \frac{\sigma_n}{\sigma_c} + \left((1 - \eta)\kappa \frac{\sigma_n}{\sigma_c} \right)^2$$

With habit formation:

$$\left(\frac{(1 - \chi)\mathcal{SR}}{\eta \sigma_c} \right)^2 = 1 - 2\rho_{c,l} \frac{\tilde{\nu}}{\tilde{\eta}} \frac{\sigma_l}{\sigma_c} + \left(\frac{\tilde{\nu}}{\tilde{\eta}} \frac{\sigma_l}{\sigma_c} \right)^2 \quad (11)$$

- Recall $\tilde{\eta} = \eta/(1 - \chi)$. Note that $\tilde{\nu} = 0$, iff $\tilde{\eta}\sigma_c = \mathcal{SR}$. This is the benchmark case of no influence of leisure on asset pricing. Note that $\tilde{\nu}$ implies $\eta = 1$, i.e. separable preferences in consumption and leisure.
- If $\eta \neq 1$, then nonseparabilities and leisure **must** play a role in the calculation of the Sharpe ratio.

Habit formation

- ❶ Uhlig, AER PP 2007, 239-243
- ❷ Kliem-Uhlig, draft, 2012
- ❸ Boldrin-Christiano-Fisher, AER 2001
- ❹ Campbell-Cochrane, 1999
- ❺ Ljunqvist-Uhlig, draft, 2012

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Non-separable utility functions

- 1 Habit formation and “catching-up-with-the-Joneses”.
- 2 **Epstein Zin.**

Epstein-Zin preferences and DSGE models

- Epstein-Zin (“EZ”) preferences: popular in asset pricing!
- Long-run risk: Bansal-Yaron, Hansen-Heaton-Li, Piazzesi-Schneider,...
- Still challenging in DSGE models.
- Tallarini, Guvenen: numerical.
- Backus-Routledge-Zin.
- Lochstoer-Kaltenbrunner

Log-linearization?

- Backus-Routledge-Zin (2007): *“If the procedure is straightforward, the calculations are not. See Appendix B for the gruesome details. ... [Appendix B:] This is a complete mess; its essential feature for our purposes is that all of these coefficients are linear functions of the value function parameters.*
- Goal in Uhlig, “Easy EZ in DSGE” (draft, 2010): make it (comparatively) **easy** to use Epstein-Zin (“**EZ**”) preferences in dynamic stochastic general equilibrium (“**DSGE**”) models.

Preferences

Growth consistent. Thus,

$$V_t = \left((1 - \tilde{\beta}) (c_t \Phi(n_t))^{1-\rho} + \beta \mathcal{R}_t^{1-\rho} \right)^{\frac{1}{1-\rho}}$$

where $\rho > 0$, $\rho \neq 1$, $0 < \beta < 1$, $0 < \tilde{\beta} < 1$ and where

$$\mathcal{R}_t = \left(E_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}$$

Note: **labor matters!**

Impact of Epstein-Zin on dynamics

1 Via adjustment in dynamics due to risk premia.

- ▶ With (steady state) risk premium: $\overline{R^{(k)}} > 1/\tilde{\beta}$.
- ▶ Capital return dynamics,

$$\hat{R}_t^{(k)} = -\hat{q}_{t-1} + \frac{\bar{r}}{\overline{R^{(k)}}} \hat{r}_t + \frac{\zeta}{\overline{R^{(k)}}} \hat{q}_t$$

- ▶ This matters for calculating \bar{k}/\bar{y} and thus for \bar{x}/\bar{y} , \bar{c}/\bar{y} .
- ▶ Similarly for safe return, $\overline{R^{(b)}} < 1/\tilde{\beta}$. Matters for government debt evolution.
- ▶ Effect: small.

2 Via stochastic discount factor \hat{M}_{t+1} :

$$0 = E_t \left[\hat{\mathbf{M}}_{t+1} + \hat{R}_{t+1} \right]$$

Note: only **predictable** movements in \hat{M}_{t+1} matter!

The value function

- Original:

$$\begin{aligned} V_t^{1-\rho} &= (1-\beta)(c_t \Phi(n_t))^{1-\rho} + \beta \tilde{\mathcal{R}}_t^{1-\rho} \\ \tilde{\mathcal{R}}_t &= \left(E_t \left[(V_{t+1})^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}} \end{aligned}$$

- Log-linearized:

$$\begin{aligned} \hat{V}_t &= (1-\beta)(\hat{c}_t - \kappa \hat{n}_t) + \tilde{\beta} \hat{\mathcal{R}}_t \\ \hat{\mathcal{R}}_t &= E_t \left[\hat{V}_{t+1} \right] \end{aligned}$$

The stochastic discount factor

- Original, detrended:

$$M_{t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\rho} \left(\frac{\Phi(n_{t+1})}{\Phi(n_t)} \right)^{1-\rho} \left(\frac{V_{t+1}}{\mathcal{R}_t} \right)^{\rho-\gamma}$$

- Log-linearized:

$$\begin{aligned} \hat{M}_{t+1} = & -\rho (\hat{c}_{t+1} - \hat{c}_t) - (1 - \rho)\kappa (\hat{n}_{t+1} - \hat{n}_t) \\ & + (\rho - \gamma) (\hat{V}_{t+1} - \hat{\mathcal{R}}_t) \end{aligned}$$

Epstein-Zin, loglinearized: Summary

$$\hat{V}_t = (1 - \beta)(\hat{c}_t - \kappa \hat{n}_t) + \beta \hat{\mathcal{R}}_t$$

$$\hat{\mathcal{R}}_t = E_t[\hat{\mathbf{V}}_{t+1}]$$

$$\begin{aligned} \hat{M}_{t+1} = & -\rho(\hat{c}_{t+1} - \hat{c}_t) - (1 - \rho)\kappa(\hat{n}_{t+1} - \hat{n}_t) \\ & + (\rho - \gamma)(\hat{\mathbf{V}}_{t+1} - \hat{\mathcal{R}}_t) \end{aligned}$$

Note:

$$E_t[\hat{M}_{t+1}] = -\rho(E_t[\hat{c}_{t+1}] - \hat{c}_t) - (1 - \rho)\kappa(E_t[\hat{n}_{t+1}] - \hat{n}_t)$$

Adding EZ to your DSGE model

- Suppose you have a (log-)linearized DSGE model without Epstein-Zin, but would like to add it.
- Suppose, shocks are homoskedastic (no stochastic volatility).
- Suppose, you are willing to ignore (or: know) the adjustments of the dynamic coefficients, due to risk-adjustments in $\overline{R^{(k)}}$, $\overline{R^{(b)}}$, ...
- **Recipe:** If you are detrending with TFP growth, add

$$-(\rho - \gamma)E_t[\hat{\zeta}_{t+1}]$$

to \hat{M}_{t+1} , recalculate dynamics... If not detrending: do nothing ...

- ... and you are done!
- What remains to do: characterize asset prices, given the dynamics.

Surprises

- Introduce the **surprise operator**

$$S_{t+k|t} = E_{t+k}[x] - E_t[x] \quad (12)$$

- Write S_{t+1} for $S_{t+1|t}$. Thus,

$$S_{t+1}[x_{t+1}] = x_{t+1} - E_t[x_{t+1}]$$

- Iteration:

$$S_{t+k|t} = S_{t+k} + S_{t+k-1} + \dots + S_{t+1} \quad (13)$$

Pricing Long-Term Risk: consumption only.

- Consider: $\rho = 1$, i.e. $\log(c) + \log \Phi(n_t)$. Suppose $\Phi(n_t) \equiv \bar{\Phi}$.
- Stochastic discount factor:

$$\begin{aligned}\hat{M}_{t+1} = & -\Delta \hat{c}_{t+1} \\ & -(\gamma - 1)S_{t+1} \left[\sum_{j=0}^{\infty} \beta^j \Delta \hat{c}_{t+j+1} \right]\end{aligned}$$

- Assume joint log-normality.
- The equity premium:

$$\begin{aligned}\log E_t[R_{t+1}] - \log R_t^f = & \text{Cov}_t \left(\hat{R}_{t+1}, \Delta \hat{c}_{t+1} \right) \\ & + (\gamma - 1) \text{Cov}_t \left(\hat{R}_{t+1}, \sum_{j=0}^{\infty} \beta^j \Delta \hat{c}_{t+1+j} \right)\end{aligned}$$

The equity premium with leisure

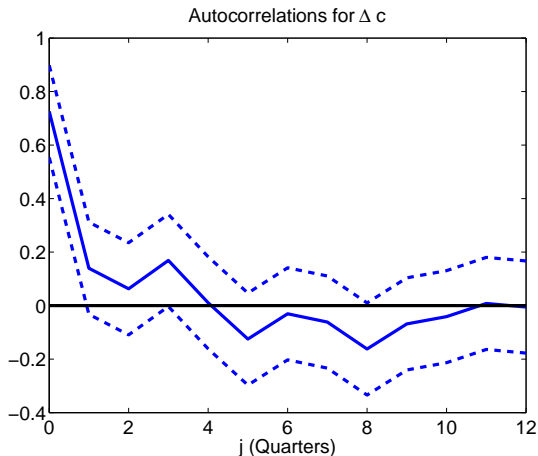
- The stochastic discount factor:

$$\begin{aligned}\hat{M}_{t+1} = & -\Delta \hat{c}_{t+1} \\ & -(\gamma - 1)S_{t+1} \left[\sum_{j=0}^{\infty} \beta^j (\Delta \hat{c}_{t+j+1} - \kappa \Delta \hat{n}_{t+j+1}) \right]\end{aligned}$$

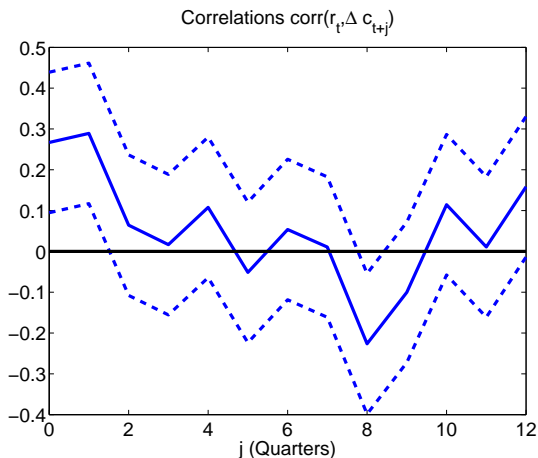
- The equity premium:

$$\begin{aligned}\log E_t [R_{t+1}] - \log R_t^f = & \text{Cov}_t \left(\hat{R}_{t+1}, \Delta \hat{c}_{t+1} \right) \\ & + (\gamma - 1) \text{Cov}_t \left(\hat{R}_{t+1}, \sum_{j=0}^{\infty} \beta^j \Delta \hat{c}_{t+1+j} \right) \\ & - (\gamma - 1) \kappa \text{Cov}_t \left(\hat{R}_{t+1}, \sum_{j=0}^{\infty} \beta^j \Delta \hat{n}_{t+1+j} \right)\end{aligned}$$

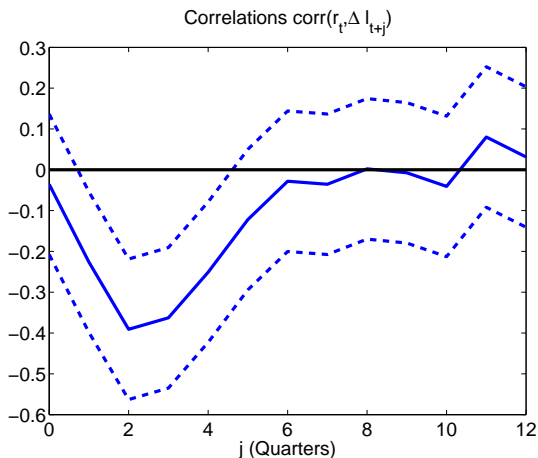
Autocorrelation of Consumption growth, Δc



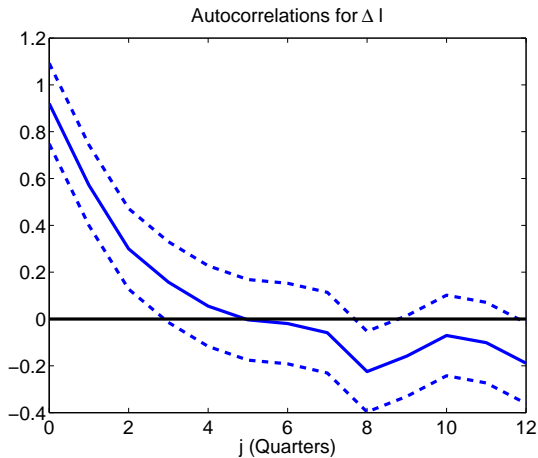
Correlations: stock returns, future first-diff. log cons.



Correlations: stock returns, future first-diff. log leisure



Autocorrelation of Leisure changes, Δl



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Disaster risk

- Rietz
- Geweke, Tsionas, Weitzman
- Barro, with co-authors
- Gabaix
- Gourio

Asset pricing and fat tails

$$\begin{aligned} 1 &= E_t[M_{t+1}R_{t+1}] \\ &= E_t[\exp(m_{t+1} + r_{t+1})] \end{aligned}$$

$$\text{Let } x_{t+1} = m_{t+1} + r_{t+1}$$

- If $x_{t+1} \sim \mathcal{N}(\mu, \sigma^2)$, rhs equals $\exp(\mu + \sigma^2/2)$.
- If $x_{t+1} \sim t_n(\mu, \sigma^2)$, rhs equals ∞ (or non-convergence).
- Tails matter!

Rare disasters and asset pricing



$$\log c_{t+1} = \log c_t + g + u_{t+1} + v_{t+1}$$

where $u_{t+1} \sim \mathcal{N}(0, \sigma^2)$ iid and

$$v_{t+1} = \begin{cases} 0 & \text{with prob. } 1 - p \\ \log(1 - b) & \text{with prob. } p \end{cases}$$

- b : random, power law $f(b) = A b^{-\alpha+1}$ with parameter α .
- Expected growth rate: $g^* = g + (1/2)\sigma^2 - pE[b]$.
- Preferences: CRRA with η .
- Equity: claim to consumption ("Lucas tree"). Equity premium:

$$r^e - r^f = \eta\sigma^2 + pE[b((1 - b)^{-\eta} - 1)]$$

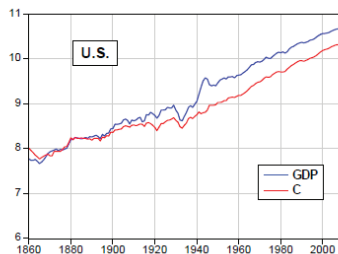
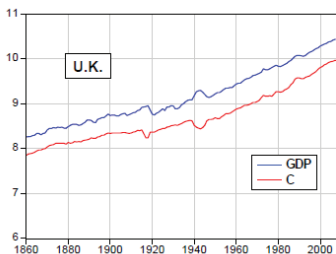
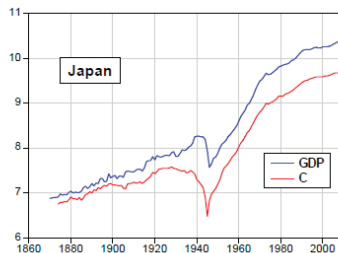
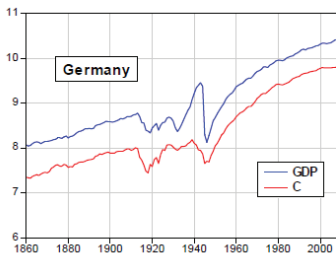
- Calibration: per $r^e - r^f = 5\%$, $\alpha = 4$, obtain $\eta = 3$.
- Further variations. E.g. Barro-Jin, double-power-law.

Measuring disasters

- 1 Barro-Ursua: new data set.
- 2 “Disaster”: decline of C by more than 10%.
- 3 Several years? Correlations? (WW II)
- 4 28 countries for C , 40 countries for GDP.
- 5 Av. disaster size: 0.216 for C , 125.
- 6 Disaster probability: 3.6% p.a..
- 7 Duration: 3.7 years.
- 8 A bit more for “GDP disasters”.

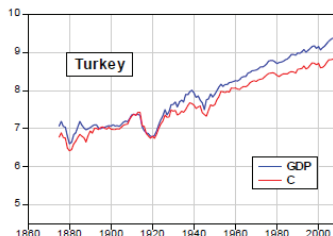
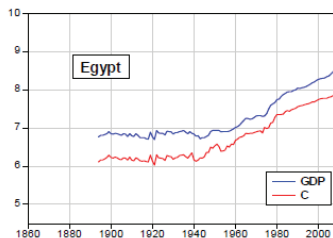
Disaster time series 1

Source: Barro slides, Barro-Ursua



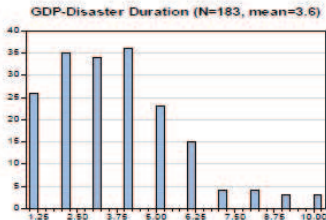
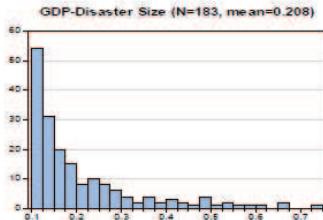
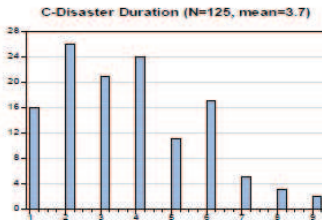
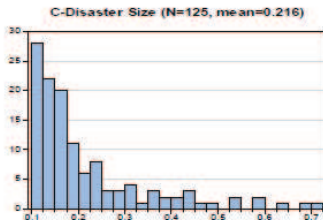
Disaster time series 2

Source: Barro slides, Barro-Ursua



Disasters: distribution

Source: Barro slides, Barro-Ursua



Disasters: table

Source: Barro slides, Barro-Ursua

| Table 4 Breakdown of Macroeconomic Crises, 1870-2006 | | | | |
|--|------------------|-----------|--------------------|-----------|
| | C (28 countries) | | GDP (40 countries) | |
| Episode/Period | Number of Events | Mean Fall | Number of Events | Mean Fall |
| Pre-1914 | 31 | 0.16 | 51 | 0.17 |
| World War I | 20 | 0.24 | 31 | 0.21 |
| Early 1920s (Flu?) | 10 | 0.24 | 8 | 0.22 |
| Great Depression | 14 | 0.20 | 23 | 0.20 |
| World War II | 21 | 0.33 | 25 | 0.37 |
| Post-World War II | 24 | 0.18 | 35 | 0.17 |
| OECD | 6 | 0.12 | 6 | 0.13 |
| Non-OECD | 18 | 0.19 | 29 | 0.17 |
| Other | 5 | 0.19 | 10 | 0.15 |
| Overall | 125 | 0.22 | 183 | 0.21 |